Physics of Motion

Velocity is the derivative of position wrt time

$$V = \frac{dx}{dt}$$

Acceleration is derivative of velocity wrt.

" is 2nd derivative of position

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Velocity is integral of acceler.

$$V = \int a\,dt$$

Position is integral of velocity

$$X = \int V\,dt$$

Position is 2nd integral of position

$$X = \iint a\,dt^2$$

A ball is thrown upwards with a velocity of 64 m/s from an initial height of 80m

a) find position as a function of time

b) when does the ball hit the ground?

$h_i = 80m \quad a = -9.8 \quad V_i = 64\ m/s$

$S(0) = 80 \quad S''(t) = -9.8 \quad S'(0) = 64$

$$\int S''(t) = \int -9.8\,dt$$

$$S'(t) = -9.8t + C$$

$$64 = -9.8 \cdot 0 + C$$

$$C = 64$$

$$\int S'(t) = \int -9.8t + 64\,dt$$

$$S(t) = -\frac{9.8t^2}{2} + 64t + C$$

$$S(t) = \tfrac{1}{2}(-9.8)t^2 + 64t + 80$$

b) when does it hit the ground?

*we want to find time when height is zero $S(t) = 0$

$$0 = -4.9t^2 + 64t + 80$$

$$t = \frac{-64 \pm \sqrt{64^2 - 4(4.9)(80)}}{2(-4.9)}$$

$$t = \frac{-64 \pm \sqrt{5664}}{-9.8}$$

$$t = \frac{-64 \pm 75.26}{-9.8} = \frac{-139.26}{-9.8}$$

$$\boxed{t = 14.21s}$$

derivation of projectile motion EQ

$$a = a$$

$$V = \int a\,dt$$

$$V = at + V_i$$

$$X = \int at + V_i\,dt$$

$$X = \frac{1}{2}at^2 + V_i t + X_i$$

$$X_f - X_i = \frac{1}{2}at^2 + V_i t$$

$$\boxed{\Delta X = \frac{1}{2}at^2 + V_i t}$$

c) What is the maximum height of the ball?

$s(t) = \frac{1}{2}(-9.8)t^2 + 64t + 80$

$s'(t) = -9.8t + 64 = 0$

$s''(t) = -9.8$

$-9.8t = -64$

$t = 6.53$

1) find critical points by setting $s'(t) = 0$

2) plug t into $s(t)$

max height $= -4.9(6.53)^2 + 64(6.53) + 80$

$= \boxed{289\text{m}}$

d) How fast is the ball moving when it hits the ground?

$s(t) = \frac{1}{2}(-9.8)t^2 + 64t + 80$ $\quad t = 14.21\text{s}$

$s'(t) = -9.8t + 64 = -9.8(14.21) + 64$

$s''(t) = -9.8$

$\boxed{V = -75.26\ \text{m/s}}$