Summation Rules

$$\sum_{i=1}^{n} c = nc$$

$$\sum_{i=1}^{4} c = 4c \qquad \sum_{i=1}^{15} c = 15c$$

$$\sum_{i=1}^{5} i = 1+2+3+4+5$$

$$\sum_{i=1}^{n} i = 1+2+3+\cdots+(n-2)+(n-1)+n$$

$$\sum_{i=1}^{n} i = n+(n-1)+(n-2)+\cdots+3+2+1$$

$$2\left(\sum_{i=1}^{n} i\right) = \underbrace{(n+1)+(n+1)+(n+1)\cdots+(n+1)+(n+1)+(n+1)}_{n \text{ terms}}$$

$$2\left(\sum_{i=1}^{n} i\right) = n(n+1)$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} c = nc$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{i=1}^{n} c(\text{thing}) = c\sum_{i=1}^{n} \text{thing}$$

$$\sum_{i=1}^{n}(a \pm b) = \sum_{i=1}^{n} a \pm \sum_{i=1}^{n} b$$

$$\sum_{i=1}^{50} 4i + 3i^2$$

$$4\sum_{i=1}^{50} i + 3\sum_{i=1}^{50} i^2$$

$$4\,\frac{n(n+1)}{2} + 3\,\frac{n(n+1)(2n+1)}{6}$$

$$\frac{4(50)(51)}{2} + \frac{3(50)(51)(101)}{6}$$

$$\boxed{133875}$$

$$\sum_{i=1}^{15} i(i+3)^2 = i(i^2+6i+9)$$

$$i^3+6i^2+9i$$

$$\frac{n^2(n+1)^2}{4} + \cancel{6}\,\frac{n(n+1)(2n+1)}{\cancel{6}} + \frac{9n(n+1)}{2}$$

$$\frac{15^2(16)^2}{4} + 15(16)(31) + \frac{9(15)(16)}{2}$$

$$\boxed{22920}$$

$$\sum_{i=1}^{n} (2i+1)^2 = 4i^2+4i+1$$

$$4\frac{n(n+1)(2n+1)}{6} + 4\frac{n(n+1)}{2} + 1\cdot n$$

$$n=8$$

$$\frac{4(8)(9)(17)}{6} + \frac{4(8)(9)}{2} + 8$$

$$\boxed{968}$$