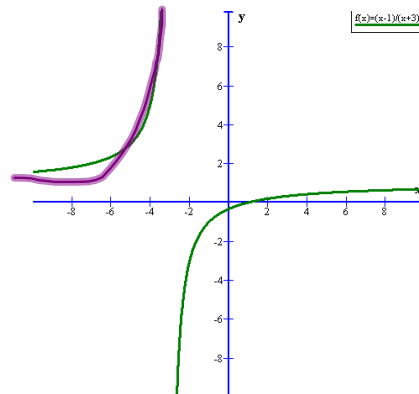


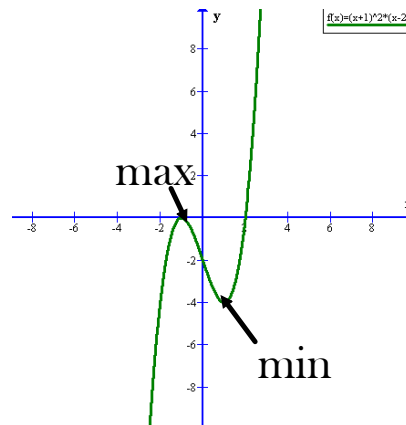
1. Find all intervals on which the graph of $f(x) = \frac{x-1}{x+3}$ is concave upward

- a. $(-\infty, \infty)$
- b. $(-\infty, -3)$
- c. $(1, \infty)$
- d. $(-3, \infty)$
- e. None of these



2. Find the values of x that give relative extrema for the function $f(x) = (x+1)^2(x-2)$

- a. Relative maximum: $x = -1$; relative minimum: $x = 1$
- b. Relative maxima: $x = 1, x = 3$; relative minimum $x = -1$
- c. Relative minimum: $x = 2$
- d. Relative maximum $x = -1$; relative minimum $x = 2$
- e. None of these



3. Let $f''(x) = 3x^2 - 4$ and let $f(x)$ have critical numbers $-2, 0, 2$. Use the Second Derivative Test to determine which critical numbers, if any, give a relative maximum.

- a. -2
- b. 2
- c. 0
- d. -2 and 2
- e. None of these

$$f''(-2) = +$$

$$f''(0) = -$$

$$f''(+2) = +$$

4. Find all points of inflection of $f(x) = \frac{1}{12}x^4 - 2x^2 + 15$

- ~~a. (2, 0)~~
- ~~b. (2, 0), (-2, 0)~~
- ~~c. (0, 15)~~
- d. $(2, \frac{25}{3}), (-2, \frac{25}{3})$
- ~~e. None of these~~

$$f'(x) = \frac{1}{3}x^3 - 4x$$

$$f''(x) = x^2 - 4 = 0$$

$$x^2 = 4$$

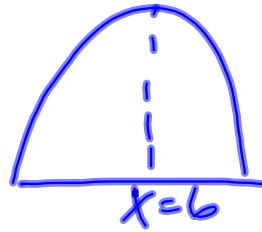
$$x = \pm 2$$

$$\frac{1}{12}2^4 - 2(4) + 15$$

$$\frac{16}{12} - 8 + 15 \neq 0$$

5. Given that $f(x) = -x^2 + 12x - 28$ has a relative maximum at $x=6$, choose the correct statement

- ~~a. f' is negative on the interval $(-\infty, 6)$~~
- ~~b. f' is positive on the interval $(-\infty, \infty)$~~
- c. f' is negative on the interval $(6, \infty)$
- ~~d. f' is positive on the interval $(6, \infty)$~~
- ~~e. None of these~~



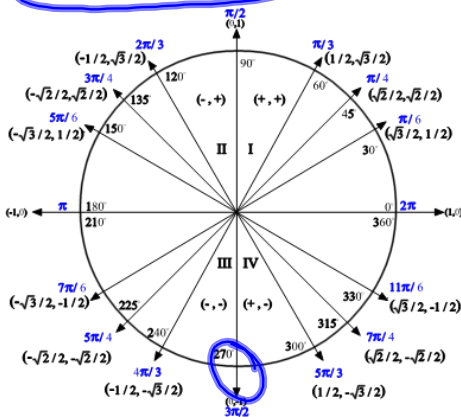
6. Find all extrema in the interval $[0, 2\pi]$ for $y = x - \cos x$

- a. $x = \frac{3\pi}{2}, 2\pi$
- b. $x = \pi, 0$
- c. $x = 2\pi, 0$
- d. $x = 2\pi, \pi$
- e. None of these

$$y' = 1 + \sin x = 0$$

$$\sin x = -1$$

$$x = \frac{3\pi}{2}$$



7. Let $f(x)$ be a polynomial function such that $f(-2) = 5$, $f'(-2) = 0$, and $f''(-2) = 3$. What is the point $(-2, 5)$.

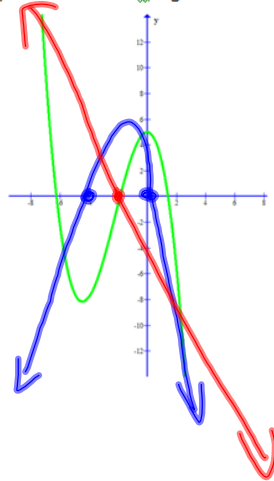
- a. Relative minimum
- b. Relative maximum
- c. X-Intercept
- d. Point of Inflection
- e. None of these

↓
min or max

↓
concave up



8. The graph of the polynomial function f is given. On the same coordinate axis sketch f' and f'' .



9. Find two nonnegative numbers whose sum is 9 and so that the product of one number and the square of the other number is a maximum.

$$x + y = 9 \quad xy^2 = \max$$

$$x = 9 - y \quad (9 - y)y^2 = 9y^2 - y^3$$

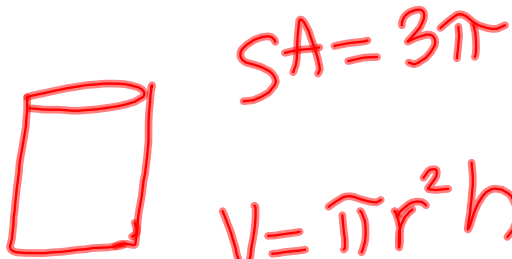
$$18y - 3y^2 = 0$$

$$3y(6 - y) = 0$$

$$y = 0 \quad y = 6$$

$$\boxed{x = 3 \quad y = 6}$$

10. A container in the shape of a right circular cylinder with no top has surface area 3π ft.² What height h and base radius r will maximize the volume of the cylinder?



$$SA = \pi r^2 + 2\pi r h = 3\pi$$

$$r^2 + rh = 3$$

$$h = \frac{3 - r^2}{r}$$

$$V = \pi r^2 \left(\frac{3 - r^2}{r} \right)$$

$$V = 3\pi r - \pi r^3$$

$$V' = 3\pi - 3\pi r^2 = 0$$

$$3\pi = \pi + 2\pi h$$

$$2\pi = 2\pi h$$

$$h = 1$$

$$3\pi = 3\pi r^2$$

$$r^2 = 1$$

$$r = \pm 1$$

$r = 1$	$h = 1$
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11. Find the point (x, y) on the graph of $y = \sqrt{x}$ nearest the point $(4, 0)$.

$$d = \sqrt{(x-4)^2 + (y-0)^2}$$

$$d = \sqrt{(x-4)^2 + (\sqrt{x})^2}$$

$$d = \sqrt{x^2 - 8x + 16 + x}$$

$$d = \sqrt{x^2 - 7x + 16}$$

$$d' \approx 2x - 7 = 0 \quad \left(\frac{7}{2}, \sqrt{\frac{7}{2}} \right)$$
$$x = \frac{7}{2}$$

12. There are 50 apple trees in an orchard. Each tree produces 800 apples. For each additional tree planted in the orchard, the output per tree drops by 10 apples. How many trees should be added to the existing orchard in order to maximize the total output of trees?

$$\text{Output} = (\# \text{ of trees}) (\text{output per tree})$$

$$O = (50 + x)(800 - 10x)$$

$$\begin{aligned} O &= 40000 - 500x + 800x - 10x^2 \\ &= 40000 + 300x - 10x^2 \end{aligned}$$

$$O' = 300 - 20x = 0$$

$$20x = 300$$

$$x = \frac{300}{20} = 15$$

They should add 15 trees