Integration by parts

$\int x \ln x \, dx \quad \int e^x \sin x \, dx$

$\int x \cos x \, dx \quad \int x^2 e^x \, dx$

$$\frac{d}{dx} uv = u \cdot \frac{dv}{dx} + v \frac{du}{dx}$$

$$\int \frac{d}{dx} uv = \int uv' + vu'$$

$$uv = \int uv' dx + \int vu' dx$$

$$uv = \int u\,dv + \int v\,du$$

$$-\int v\,du$$

$$uv - \int v\,du = \int u\,dv$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

$$\int u\,dv = uv - \int v\,du$$

$$\int x \sin x\,dx$$

$u = x \qquad du = dx$

$dv = \sin x\,dx$

$v = -\cos x$

$$(x)(-\cos x) - \int -\cos x\,dx$$

$$\boxed{-x\cos x + \sin x + C}$$

$$\int x^2 \ln x \, dx$$

$u = \ln x \qquad du = \frac{dx}{x}$

$dv = x^2 dx \qquad v = \frac{x^3}{3}$

$$\ln x \left(\frac{x^3}{3}\right) - \int \frac{x^{\cancel{3}\,2}}{3} \cdot \frac{dx}{\cancel{x}}$$

$$\ln x \left(\frac{x^3}{3}\right) - \frac{1}{3} \int x^2 dx$$

$$\ln x \left(\frac{x^3}{3}\right) - \frac{1}{3} \cdot \frac{x^3}{3} + C$$

$$\boxed{\frac{x^3}{3}\left(\ln x - \frac{1}{3}\right) + C}$$

$$\int \ln x \, dx$$

$u = \ln x \qquad du = \frac{dx}{x}$

$dv = dx \qquad v = x$

$$x\ln x - \int \cancel{x}\frac{dx}{\cancel{x}}$$

$$x\ln x - \int dx$$

$$x\ln x - x + C$$

$$\int \ln x = x\ln x - x + C$$

$$\int (\ln x)^2 dx$$

$$u = \ln x \qquad du = \frac{dx}{x}$$

$$dv = \ln x\, dx \qquad v = x\ln x - x$$

$$\ln x(x\ln x - x) - \int (x\ln x - x)\frac{dx}{x}$$

$$\ln x(x\ln x - x) - \int (\ln x - 1)\, dx$$

$$\ln x(x\ln x - x) - \int \ln x\, dx - \int 1\, dx$$

$$\ln x(x\ln x - x) - x\ln x + x - x \quad + C$$

$$\ln x(x\ln x - x) - x\ln x$$

$$x(\ln x)^2 - x\ln x - x\ln x + C$$

$$x(\ln x)^2 - 2x\ln x + C$$

$$x\ln x(\ln x - 2) + C$$